

HILLSBOROUGH TOWNSHIP SCHOOL DISTRICT

MATHEMATICS CURRICULUM

Math Analysis

July, 2020

Course Overview

Math Analysis

The Math Analysis curriculum is written to address the needs of students in the College Preparatory and Honors classes. This course is structured around the New Jersey Student Learning Standards.

Topics that will be covered include linear, polynomial, rational, exponential, logarithmic and trigonometric functions and their graphs. Students will also be exposed to analytic trigonometry, matrices, vectors, series and sequences, and an introduction to calculus.

A variety of tools and strategies will be incorporated into the curriculum to enhance the learning of every child. Digital tools are part of the textbook resources.

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

Unit Title: Unit 1 Foundations	Timeframe/Pacing: 8 days
Essential Questions <ul style="list-style-type: none"> ● How are all the parts of mathematics related? ● What makes an appropriate solution? ● How can we relate skills learned previously? 	
Enduring Understandings <ul style="list-style-type: none"> ● The solutions to an equation are one or more values. ● The solutions to an inequality are sets of values. 	
Standards Taught and Assessed <ul style="list-style-type: none"> ● A-REI.B. Solve equations and inequalities in one variable <ol style="list-style-type: none"> 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. 4. Solve quadratic equations in one variable. <ol style="list-style-type: none"> a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. 	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none"> ● ELA: SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one on- one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively. 	
Highlighted Career Ready Practices and 21st Century Themes and Skill <ul style="list-style-type: none"> ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 	
Social Emotional Learning Competencies <ul style="list-style-type: none"> ● 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations. ● 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity} 	

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

Pre-Assessment <ul style="list-style-type: none"> Pre-test of Algebra 2 skills 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
<input checked="" type="checkbox"/> 1. I can use set notation and find the union and intersection of sets.	SMP 5 Use appropriate tools strategically.	Find $A \cup B$ and $A \cap B$ where: $A = \{2, 4, 6, 8, 10\}$ and $B = \{1, 2, 3, 4, 5\}$	Large group discussion, connect to ideas of "or" vs "and"	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
<input checked="" type="checkbox"/> 2. I can solve linear and absolute value equations.	SMP 7 Look for and make use of structure.	Solve $ 4x + 9 = 5$	Small group practice, students present solutions	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
<input checked="" type="checkbox"/> 3. I can solve a formula for one variable.	SMP 6 Attend to precision.	For a given equation, for example $\frac{1}{x} + \frac{1}{y} = 10$, solve for x.	Teacher led discussion, small group practice, connect to science and needing to isolate different variables.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.

Key: Honors and CP Honors/CP optional Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

				504 plan.
<input type="checkbox"/> 4. I can solve linear inequalities. (for Honors: add absolute value inequalities).	SMP 7 Look for and make use of structure.	Solve $ 2x + 1 < 5$ Solve $4x - 19 \geq 23$	Recall definition of absolute value, focus on distance going in two directions. Apply definition to solution method.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
<input checked="" type="checkbox"/> 5. I can use interval notation to express solutions to compound inequalities.	SMP 6 Attend to precision.	Express $3 < x \leq 10$ in interval notation	Suggest graphing on a number line and finding intervals of union or intersection.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment <ul style="list-style-type: none"> Not applicable 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Summative Assessment(s) <ul style="list-style-type: none"> Foundations - Common Assessment Unit 1 Performance Task 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		

Key: Honors and CP Honors/CP optional Honors

Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum

Unit Title: Unit 2 Functions and Graphs	Timeframe/Pacing: 18 Days
Essential Questions <ul style="list-style-type: none">• How can patterns, relations, and functions be used as tools to best describe and help explain real life situations?• What situations can be analyzed using transformations and symmetries?	
Enduring Understandings <ul style="list-style-type: none">• Algebraic representation can be used to generalize patterns and relationships. These can be expressed as functions.• Functions can be used to build other functions, Functions can be graphed and transformed.• The inverse of a function can be found and graphed. Symmetry of a function can be determined algebraically.	
Standards Taught and Assessed <ul style="list-style-type: none">• F-IF.A. Understand the concept of a function and use function notation<ol style="list-style-type: none">1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.• F-IF.B. Interpret functions that arise in applications in terms of the context<ol style="list-style-type: none">4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.• F-BF.B. Build new functions from existing functions.<ol style="list-style-type: none">3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases aa. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.<ol style="list-style-type: none">b. (+) Verify by composition that one function is the inverse of another.c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.d. (+) Produce an invertible function from a non-invertible function by restricting the domain.• F-IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	

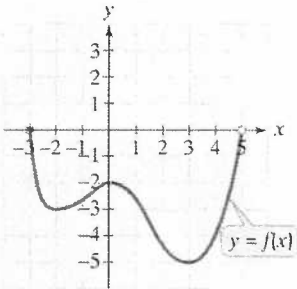
Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

Highlighted Interdisciplinary Connections				
<ul style="list-style-type: none"> • Computer Science and Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena. 				
Highlighted Career Ready Practices and 21st Century Themes and Skill				
<ul style="list-style-type: none"> • 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions • 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 				
Social Emotional Learning Competencies				
<ul style="list-style-type: none"> • 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations. • 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity} 				
Pre-Assessment		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)		
<ul style="list-style-type: none"> • How to evaluate a function • How to locate x and y intercepts. 		<ul style="list-style-type: none"> • Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
■ 1. I can interpret information given by graphs.	SMP 7 Look for and make use of structure.	Find $f(5)$ for the given function. $f(x) = 2x^2 - 3x + \sqrt{4x - 4}$	Using online textbook materials, videos, teacher prepared notes, and exercises.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>■ 2. I can identify domain and range of a function expressed in various representations (table, graph, equation, words).</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Determine the domain and range of</p> <p>a) $f(x) = 3 x - 2 + 1$</p> <p>b)</p> <table border="1" data-bbox="1029 358 1236 751"> <thead> <tr> <th>x</th> <th>h(x)</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>0</td> </tr> <tr> <td>-1</td> <td>.75</td> </tr> <tr> <td>0</td> <td>3</td> </tr> <tr> <td>4</td> <td>-1</td> </tr> <tr> <td>5</td> <td>5</td> </tr> </tbody> </table>	x	h(x)	2	0	-1	.75	0	3	4	-1	5	5	<p>Using online textbook materials, videos, teacher prepared notes, and exercises.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
x	h(x)															
2	0															
-1	.75															
0	3															
4	-1															
5	5															
<p>■ 3. I can identify intervals where a function is increasing or decreasing.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Determine the intervals for which the function is increasing or decreasing:</p> 	<p>Using online textbook materials, videos, teacher prepared notes, and exercises. By examination of graphs.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>												
<p>■ 4. I can determine whether a relation is a function.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Determine whether the equation defines y as a function of x:</p>	<p>Using online textbook materials, videos, teacher prepared notes, and</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications</p>												

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

			exercises. Specifically using the vertical line test.	ions per a student's IEP or 504 plan.
<p>■ 5. I can identify a function as even or odd and recognize symmetry. (Honors only: Perform tests of symmetry: x-axis, y-axis, origin, $y=x$)</p>	SMP 7 Look for and make use of structure.	Determine whether the given function is even, odd, or neither for $f(x) = x^3 - 5x$	Using online textbook materials, videos, teacher prepared notes, and exercises. Evaluate given function symbolically for positive or negative x or y .	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
<p>■ 6. I can find and simplify a function's difference quotient.</p>	SMP 6 Attend to precision.	Find and simplify the difference quotient for $f(x) = 8x - 11$	Carefully evaluate functions $f(x+h)$ and evaluate difference quotient	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
<p>■ 7. I can write the equation of a line in slope-intercept form.</p>	SMP 7 Look for and make use of structure.	Given a point $(5, -3)$ on line L parallel to $2x - 4y + 10 = 0$ write the equation for L in slope-intercept form.	Using online textbook materials, videos, teacher prepared notes, and exercises.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>■ 8. I can identify x and y intercepts given an equation.</p>	<p>SMP 6 Attend to precision.</p>	<p>Find the x and y intercepts of $f(x) = x^2 - 5x + 6$</p>	<p>Using online textbook materials, videos, teacher prepared notes, and exercises.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 9. I can find the average rate of change of a function.</p>	<p>SMP 6 Attend to precision.</p>	<p>Find the average rate of change of $f(x) = x^2 - 4x$ from $x_1 = 5$ to $x_2 = 9$</p>	<p>Compare to a familiar quantity, such as the temperature over time.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 10. I can identify the parent function and transformations for a given function.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Graph the parent function and use transformations to graph $f(x) = .5(x - 1)^2 + 1$</p>	<p>Use Desmos with sliders for powerful demonstration of the transformation of function.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 11. I can recognize graphs of common parent functions and identify their domain and range.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find the domain and range of $y = \sqrt{8 - 2x}$.</p>	<p>Using online textbook materials, videos, teacher prepared notes, and exercises.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>■ 12. I can add, subtract, multiply and form composite functions.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Given $f(x) = x^2 + 3$ and $g(x) = 4x - 1$, find f of g.</p>	<p>Using online textbook materials, videos, teacher prepared notes, and exercises.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 13. I can find and verify inverse functions.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find the inverse of $f(x) = 8x^3 - 1$</p>	<p>Using online textbook materials, videos, teacher prepared notes, and exercises.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>⊙ 14. I can identify periodicity and amplitude of a periodic function, and the effect transformations have on them.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>If the period of $g(x)$ is 6, what is the period of $f(2x)$?</p>	<p>Using online textbook materials, videos, teacher prepared notes, and exercises.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>Benchmark Assessment</p> <ul style="list-style-type: none"> ● Not applicable 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
<p>Summative Assessment(s)</p> <ul style="list-style-type: none"> ● Functions and Graphs - Common Assessment ● Unit 2 Performance Task 				

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum

Unit Title: Unit 3 Polynomial and Rational Functions	Timeframe/Pacing: 23 days
Essential Questions <ul style="list-style-type: none">● What makes an algebraic algorithm both effective and efficient?● How can we use mathematical models to describe physical relationships?● How are patterns of change related to the behavior of functions?	
Enduring Understandings <ul style="list-style-type: none">● Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole.● Mathematical models can be used to describe and quantify physical relationships.● Patterns and relationships can be represented graphically, numerically, symbolically, or verbally.	
Standards Taught and Assessed <ul style="list-style-type: none">● F-IF.C. Analyze functions using different representations.<ul style="list-style-type: none">7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.<ul style="list-style-type: none">c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.● N-CN. C. Use complex numbers in polynomial identities and equations.<ul style="list-style-type: none">7. Solve quadratic equations with real coefficients that have complex solutions.8. (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.● A-APR. B. Understand the relationship between zeros and factors of polynomials<ul style="list-style-type: none">2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.● A-APR D. Rewrite rational expressions<ul style="list-style-type: none">6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>Highlighted Interdisciplinary Connections</p> <ul style="list-style-type: none"> ● Computer Science & Design Thinking: 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena 				
<p>Highlighted Career Ready Practices and 21st Century Themes and Skill</p> <ul style="list-style-type: none"> ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 				
<p>Social Emotional Learning Competencies</p> <ul style="list-style-type: none"> ● 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations. ● 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity} 				
<p>Pre-Assessment</p> <ul style="list-style-type: none"> ● How to factor polynomials ● Add, subtract, multiply polynomials 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student’s IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
<p>■ 1. I can perform operations on imaginary numbers.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Simplify $\frac{3+2i}{4-i}$</p>	<p>Connect to simplifying roots</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student’s IEP or 504 plan.</p>

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>■ 2. I can locate the min or max of a quadratic function.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find the min or max of $f(x) = -x^2 + 4x - 3$</p>	<p>Utilize both forms (vertex form and standard form), connect to solving word problems</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 3. I can determine end behavior of a polynomial and rational function.</p>	<p>SMP 4 Model with mathematics.</p>	<p>Determine the end behavior of $f(x) = -x^3 + 5x^2 + 3$</p>	<p>Examine simple examples to develop patterns of end behavior, dynamic graphing</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 4. I can locate x-intercepts of a polynomial function and determine behavior near the x-intercepts.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find the zeros and determine the behavior of $f(x) = x^6 - 6x^4 + 9x^2$</p>	<p>Connect structures of equations to process/strategy of solving, dynamic graphing connection, teacher led discussion</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 5. I can perform synthetic substitution, division, and long division of polynomials.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find $f(5)$ for the given function. $f(x) = 5x^4 + 2x^2 - 3x - 3$</p>	<p>Connect table of value/graph/equation with the solution, small group exercises.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 6. I can apply the Rational Zero Theorem to</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find all possible zeros for $f(x) = -2x^3 - 6x^2 + 18x$</p>	<p>Connect actual zeros to structures of polynomials, relate theorem to zeros,</p>	<p>Use of calculator, extended time, and specific other</p>

Key: ■ Honors and CP □ Honors/CP optional ◎ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

determine any and all rational zeros.			small group sets of scaled examples	accommodations/modifications per a student's IEP or 504 plan.
■ 7. I can identify horizontal/vertical/slant asymptotes of a function.	SMP 7 Look for and make use of structure.	Find any and all asymptotes for $f(x) = \frac{x^2-4x-32}{x-3}$	Relate end behaviors to structures in equation, connect table/graph/equation to one another, dynamic graphing, teacher led discussion	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 8. I can solve a polynomial equation using various methods.	SMP 8 Look for and express regularity in repeated reasoning.	Solve $f(x) = -2x^3 - 6x^2 + 18x$	Small group investigation/conversations, scaled examples/exercises, compare methods of solving	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
□ 9. I can solve a polynomial inequality and express the solution in interval notation, set notation, and on a number line. (Honors: Solve rational inequalities, graph two variable polynomial inequalities)	SMP 4 Model with mathematics.	Solve $(x + 3)(x + 2)(x - 1) > 0$.	Relate structure of expression/graph of equation/table of values with solution set, teacher led investigation, scaled examples with small group exercises.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum

<p>■ 10. I can graph polynomial functions.</p>	<p>SMP 4 Model with mathematics.</p>	<p>Graph $f(x) = -2x^3 - 6x^2 + 18x$</p>	<p>Connect to zeros and end behavior, dynamic graphing, group discussion to predict behavior</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 11. I can graph rational functions.</p>	<p>SMP 4 Model with mathematics.</p>	<p>Graph $f(x) = \frac{x-3}{x^2-4x-5}$</p>	<p>Connect to asymptotes and intercepts, dynamic graphing, scaled group discussion and individual practice</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>□ 12. I can write a polynomial function in standard form from given conditions. (Honors: Apply sum/difference of roots, Fundamental theorem, complex conjugates).</p>	<p>SMP 4 Model with mathematics.</p>	<p>Write the cubic function where the zeros are 5 and $2 + i$.</p>	<p>Relate solutions to graphs, connect factoring to solutions, connect theorems to structures of equations, teacher led examples.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 13. I will be able to write a polynomial function given a word problem and solve for a maximum (quadratics only).</p>	<p>SMP 4 Model with mathematics.</p>	<p>A rectangle has a perimeter of 80 cm. If its width is x, express its length and its area in terms of x.</p> <p>a. Find an equation to model area as a</p>	<p>Connect to finding vertex, x intercept, and y intercept for quadratic, dynamic graphing</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

		<p>function of the width.</p> <p>b. What is the domain of A?</p> <p>c. What is the maximum area of the rectangle?</p>		
<p>Benchmark Assessment</p> <ul style="list-style-type: none"> ● Quarterly (Unit 2, Unit 3) (for Honors) ● Not applicable for CP 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
<p>Summative Assessment(s)</p> <ul style="list-style-type: none"> ● Polynomial and Rational Functions - Common Assessment ● Unit 3 Performance Task 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum

Unit Title: Unit 4 Exponential and Logarithmic Functions	Timeframe/Pacing: 16 days
<p>Essential Questions</p> <ul style="list-style-type: none"> ● What does it mean for a quantity to grow exponentially? What is the mathematical model? ● Why is the logarithmic function defined as the inverse of the exponential function? ● What type of real world problems can be solved using exponential and logarithmic functions? 	
<p>Enduring Understandings</p> <ul style="list-style-type: none"> ● Exponential functions occur naturally in the world and can be used to solve a variety of problems of growth and decay. ● The inverse logarithmic function is also useful for solving problems. . 	
<p>Standards Taught and Assessed</p> <ul style="list-style-type: none"> ● F-IF.C.7.e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ● F-IF.C.8.b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12t$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. ● F-LE.A. Construct and compare linear and exponential models and solve problems <ol style="list-style-type: none"> 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. <ol style="list-style-type: none"> a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. <ol style="list-style-type: none"> 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). 3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. 4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $abct = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. 	
<p>Highlighted Interdisciplinary Connections</p> <ul style="list-style-type: none"> ● Science: PS1.C: Nuclear Processes Spontaneous radioactive decays follow a characteristic exponential decay law. Nuclear lifetimes allow 	

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>radiometric dating to be used to determine the ages of rocks and other materials.</p> <ul style="list-style-type: none"> • Computer Science & Design Thinking: 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena 				
<p>Highlighted Career Ready Practices and 21st Century Themes and Skill</p> <ul style="list-style-type: none"> • 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions • 9.1.12.CDM.6: Compute and assess the accumulating effect of interest paid over time when using a variety of sources of credit. (e.g., student loans, credit cards, auto loans, mortgages, etc.). 				
<p>Social Emotional Learning Competencies</p> <ul style="list-style-type: none"> • 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations. • 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity} 				
<p>Pre-Assessment</p> <ul style="list-style-type: none"> • P2 Rules of Exponents • P3 Radicals and Rational exponents 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> • Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
<p>■ 1. I can graph exponential functions and their transformations.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Graph $y = 2^x$ and $y = 2^{x-1} + 3$</p>	<p>Dynamic graphing, connect structures in equation to graphs, small group exercises</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

■ 2. I can graph logarithmic functions and their transformations.	SMP 7 Look for and make use of structure.	Graph $y = \log_3 x$ and $y = -\log_3 x + 1$	Dynamic graphing, connect structures in equation to graphs, small group exercises	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 3. I can evaluate logarithms.	SMP 6 Attend to precision.	Evaluate $\log_4 4^{2x}$ and $\log_3(1/9)$	Relate multiple forms, connect properties of exponents, small group practice	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 4. I can find the domain of logarithmic functions.	SMP 7 Look for and make use of structure.	Find the domain of $f(x) = \log(1-x) + 2$.	Connect structure of expression to solution, dynamic graphing	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 5. I can expand and condense logarithmic expressions.	SMP 7 Look for and make use of structure.	Condense the logarithm: $.5\log x - \log(y + 1) + \log 10$	Relate multiple forms, connect properties of exponents, small group practice	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 6. I can solve exponential equations.	SMP 6 Attend to precision.	Solve $3^{x+4} = 2^{x-1}$	Connect to properties of exponents/logs, relate patterns of structures in	Use of calculator, extended time, and specific other

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

			equations to process, teacher guided examples	accommodations/modifications per a student's IEP or 504 plan.
■ 7. I can solve logarithmic equations.	SMP 6 Attend to precision.	Solve $\log(x + 1) - \log(2x+3) = \log 4$	Connect to properties of exponents/logs, relate patterns of structures in equations to process, teacher guided examples	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 8. I can apply exponential and logarithmic rules to solve application problems.	SMP 6 Attend to precision.	The half life of strontium-90 is 28 days. Suppose that a nuclear accident occurs and releases 60 grams of strontium-90 into our atmosphere. How long will it take for the strontium-90 to decay to a level of 10 grams?	Small group practice, connect parts of problem to structures in equations, relate graph to solution.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment <ul style="list-style-type: none"> Units 2 - 4 Cumulative Assessment (CP only) Not applicable for Honors 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Summative Assessment(s) <ul style="list-style-type: none"> Exponential and Logarithmic Functions - Common Assessment Unit 4 Performance Task 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum

Unit Title: Unit 5 Trigonometry Basics	Timeframe/Pacing: 15 Days
Essential Questions <ul style="list-style-type: none"> ● How can we model periodic phenomena? ● Can we extend the trigonometric relationships to consider any angle? ● What are some of the important properties of the trigonometric functions? 	
Enduring Understandings <ul style="list-style-type: none"> ● The trigonometric functions can be used to describe cyclic or periodic phenomena. ● The trigonometric definition can be extended to consider any angle (all real numbers). ● There are many interesting relationships between and properties of the trigonometric functions. 	
Standards Taught and Assessed <ul style="list-style-type: none"> ● F-TF.A. Extend the domain of trigonometric functions using the unit circle <ol style="list-style-type: none"> 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. 2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. 3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for πx, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. 4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. 	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none"> ● Science: HS-PS4-1 Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media {sinusoidal waves} ● Visual & Performing Arts: 1.3E.12prof.Cr1a: Generate melodic, rhythmic and harmonic ideas for compositions or improvisations using digital tools. 	
Highlighted Career Ready Practices and 21st Century Themes and Skill <ul style="list-style-type: none"> ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 	
Social Emotional Learning Competencies <ul style="list-style-type: none"> ● 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations. 	

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<ul style="list-style-type: none"> 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity} 				
Pre-Assessment <ul style="list-style-type: none"> Right triangle trigonometric relationships. 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
<p>■ 1. I can find positive and negative coterminal angles for a given angle.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find a positive and negative angle that is coterminal with angle $-5\pi/12$.</p>	<p>Learn to measure angles and relationship between coterminal angles.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 2. I can find all six trigonometric functions when given one or two functions.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find all six trigonometric functions given the following: $\sin\theta = 8/17$ and $\cos\theta = 15/17$. Find all six trigonometric functions given the following information: $\cos\theta = 1/3$ and θ is in quadrant IV.</p>	<p>Learn the unit circle definitions of the trigonometric functions and their relationships</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>■ 3. I can find all six trigonometric functions when given a point.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find all six trigonometric functions given the point $P(3,7)$ is on the terminal side of angle θ.</p>	<p>Extend definitions of trigonometric functions beyond the unit circle (finding the radius)</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 4. I can evaluate all six trigonometric functions when given a common angle.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find all six trigonometric functions for $\theta = \pi/6$.</p>	<p>Find and KNOW trig function values using special right triangles. Play unit circle online games.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 5. I can find a reference angle for a given angle.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find the reference angle for -250°.</p>	<p>Discuss reference angles, show reference tables.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 6. I can solve for missing sides and angles in a right triangle.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find the missing side lengths and/or angle measures in a right triangle, given sufficient information.</p>	<p>Recall from Geometry</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>■ 7. I can solve for all six trigonometric functions given parts of a right triangle.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find all six trigonometric functions for θ, within a given right triangle.</p>	<p>Using right triangle trigonometry or inverse trig functions.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>⊙ 8. I can find arc length, sector area, and apparent size.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>The perimeter of a sector is 12 cm. It's area is 8 cm². Find the radius.</p>	<p>Connect geometric proportions, small group discovery</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan</p>
<p>Benchmark Assessment</p> <ul style="list-style-type: none"> ● Not applicable 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
<p>Summative Assessment(s)</p> <ul style="list-style-type: none"> ● Trigonometry Basics - Common Assessment ● Unit 5 Performance Task 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

Unit Title: Unit 6 Trigonometric Functions	Timeframe/Pacing: 16 days
Essential Questions <ul style="list-style-type: none"> ● How can I best represent a real-world periodic phenomena? ● How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations? ● What makes an algebraic algorithm both effective and efficient? 	
Enduring Understandings <ul style="list-style-type: none"> ● Patterns and relationships can be represented graphically, numerically, symbolically, or verbally. ● Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole. 	
Standards Taught and Assessed <ul style="list-style-type: none"> ● F-TF.5 - Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline ● G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ● F-TF.7 (+) - Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context ● F-TF.6 (+) - Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. ● F-IF-C.7.e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude 	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none"> ● English: SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one on- one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively. ● Computer Science: 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena ● Science: HS-PS4-1 Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media 	
Highlighted Career Ready Practices and 21st Century Themes and Skill <ul style="list-style-type: none"> ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 	

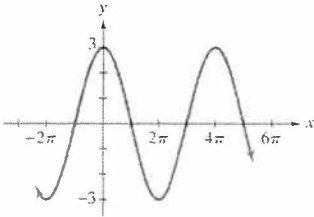
Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>Social Emotional Learning Competencies</p> <ul style="list-style-type: none"> 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations. 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity} 				
<p>Pre-Assessment</p> <ul style="list-style-type: none"> Domain/Range of trigonometric functions Even/Odd Symmetry Definition of function/inverse function 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
<p>■ 1. I can evaluate inverse trigonometric expressions.</p>	<p>SMP 6 Attend to precision.</p>	<p>Find the exact value of $\sin(\cos^{-1}(\frac{3}{5}))$</p>	<p>Scaled exercises in small groups, teacher led discussion, connect domain/range of functions, relate unit circle when appropriate</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 2. I can graph all six trigonometric functions (including finding period, amplitude, asymptotes, x-intercepts, any relative max/mins).</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Graph two periods of the function $y = -\sin(x + 2) - 5$.</p>	<p>Connect unit circles to sin/cos/tan graphs, connect sin/cos/tan to csc/sec/cot graphs, student discovery/investigation, dynamic graphing</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>■ 3. I can write an equation for a trigonometric function when given a graph.</p>	<p>SMP 4 Model with mathematics.</p>	<p>Write the function for the given graph:</p> 	<p>Connect features of graph to structures of equations, identify critical information, small group exploration</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 4. I can write an equation for a trigonometric function when given a chart or word problem.</p>	<p>SMP 4 Model with mathematics.</p>	<p>Rounded to the nearest hour, Los Angeles averages 14 hours of daylight in June, 10 hours in December, and 12 hours in March and September. Let x represent the number of months after June and let y represent the number of hours of daylight in month x. Write an equation that represents this relationship.</p>	<p>Scaled small group explorations, dynamic graphing, connect written word to structures of equation</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 5. I can solve for a missing side in a triangle when given a situation in a word problem.</p>	<p>SMP 4 Model with mathematics.</p>	<p>A hot-air balloon is rising vertically. From a point on level ground 125 feet from the point directly under the passenger compartment, the angle of elevation to the balloon changes from 19.2 degrees to 31.7 degrees. How far, to the nearest tenth of a</p>	<p>Small group discovery, scaled group problems</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

		foot, does the balloon rise during this period?		
Benchmark Assessment <ul style="list-style-type: none"> • Not applicable 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> • Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Summative Assessment(s) <ul style="list-style-type: none"> • Trigonometric Functions- Common Assessment • Unit 6 Performance Task: Ferris Wheel Lab (All) 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> • Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

Unit Title: Unit 7 Analytic Trigonometry	Timeframe/Pacing: 24 days
<p>Essential Questions</p> <ul style="list-style-type: none"> ● What makes a computational strategy both effective and efficient? ● How do operations affect numbers? ● How do mathematical ideas interconnect and build on one another to produce a coherent whole? 	
<p>Enduring Understandings</p> <ul style="list-style-type: none"> ● Computational fluency includes understanding the meaning and the appropriate use of numerical operations. ● Measurements can be used to describe, compare, and make sense of phenomena. ● Numeric fluency includes both the understanding of and the ability to appropriately use numbers. 	
<p>Standards Taught and Assessed</p> <ul style="list-style-type: none"> ● F-TF C. Prove and apply trigonometric identities <ul style="list-style-type: none"> 8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. 9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. 	
<p>Highlighted Interdisciplinary Connections</p> <ul style="list-style-type: none"> ● English: SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one on- one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively. 	
<p>Highlighted Career Ready Practices and 21st Century Themes and Skill</p> <ul style="list-style-type: none"> ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 	
<p>Social Emotional Learning Competencies</p> <ul style="list-style-type: none"> ● 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations. ● 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity} 	

Key: ■ Honors and CP □ Honors/CP optional ◎ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

Pre-Assessment <ul style="list-style-type: none"> Find the trigonometric ratios for given angle 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
■ 1. I can verify a given identity.	SMP 3 Construct viable arguments and critique the reasoning of others.	Verify $\frac{\cos x}{1-\sin x} - \frac{\cos x}{1+\sin x} = 2\tan x$	Connect with geometry proofs, focus on multiple methods, small group practice to see different ways	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 2. I can find exact values using sum or difference of angles when given the angle. (Honors: add can simplify and evaluate expressions using sum or difference)	SMP 5 Use appropriate tools strategically.	Find the exact value of $\cos(75^\circ)$.	"Prove" using examples, relate to unit circle, small group practice	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 3. I can find exact values using sum or difference of angles when given information about sine and cosine. (Honors: add given any ratio)	SMP 5 Use appropriate tools strategically.	If $\sin x = \frac{4}{5}$ and x lies in quadrant I, and $\sin y = \frac{7}{25}$ and y lies in quadrant II, find the exact value of $\cos(x+y)$, $\sin(x+y)$, and $\tan(x+y)$.	Connect to geometry, teacher guided examples	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>■ 4. I can find exact values using double angles when given information about one trigonometric function. (Honors: add half angles)</p>	<p>SMP 5 Use appropriate tools strategically.</p>	<p>If $\sin x = 5/13$, and x lies in quadrant II, find the exact value of $\sin 2x$, $\cos 2x$, and $\tan 2x$.</p>	<p>“Prove” using sum or difference formulas, teacher guided examples</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student’s IEP or 504 plan.</p>
<p>■ 5. I can solve trigonometric equations to find all solutions or solutions in a given interval. (Honors: add using sum/difference/double/half angle formulas)</p>	<p>SMP 5 Use appropriate tools strategically.</p>	<p>Solve the equation on the interval $[0, 2\pi)$. $\cos^2 x + 2\cos x - 3 = 0$.</p>	<p>Connect to inverse trigonometry and unit circle, relate with graphing of trigonometric functions and where the solutions would have been for specific y-values, small group practice</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student’s IEP or 504 plan.</p>
<p>Benchmark Assessment</p> <ul style="list-style-type: none"> ● Units 5-7 Cumulative Assessment 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student’s IEP or 504 plan. 		
<p>Summative Assessment(s)</p> <ul style="list-style-type: none"> ● Analytic Trig - Common Assessment ● Unit 7 Performance Task 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student’s IEP or 504 plan. 		

Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum

Unit Title: Unit 8 Additional Trigonometry	Timeframe/Pacing: 15 days
Essential Questions <ul style="list-style-type: none">● How do mathematical ideas interconnect and build on one another to produce a coherent whole?● How can measurements be used to solve problems?● In what different ways can we represent quantities?● In what different ways can we represent and visualize questions?	
Enduring Understandings <ul style="list-style-type: none">● Numeracy includes both the understanding of and the ability to appropriately use numbers.● Everyday objects have a variety of attributes, each of which can be measured in many ways.● Complex numbers can be represented algebraically and visually in different forms.● Vectors can be used to represent quantities of magnitude and directions.	
Standards Taught and Assessed <ul style="list-style-type: none">● G-SRTT D. Apply trigonometry to general triangles.<ul style="list-style-type: none">9. (+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).● N-VM A. Represent and model with vector quantities.<ul style="list-style-type: none">1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, v, $\ v\$, v).2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.● N-VM B. Perform operations on vectors.<ul style="list-style-type: none">4. (+) Add and subtract vectors.<ul style="list-style-type: none">a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector	

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum

subtraction component-wise.

5. (+) Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

b. Compute the magnitude of a scalar multiple cv using $\|cv\| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

- N-CN.B. Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and an argument 120° .

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Highlighted Interdisciplinary Connections

- English: SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one on- one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively.
- Science: HS-PS2-2 Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations.
- 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity}

Key: ■ Honors and CP □ Honors/CP optional ◎ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

Pre-Assessment <ul style="list-style-type: none"> Solve right triangles 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
■ 1. I can solve triangles using the Law of Sines.	SMP 5 Use appropriate tools strategically.	Solve the triangle with $A = 40^\circ$, $a = 54$, and $b = 62$.	Teacher led practice, then small group practice, focus on when to use law of sines versus cosines	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 2. I can solve triangles using the Law of Cosines.	SMP 5 Use appropriate tools strategically.	Solve the triangle with $a = 6$, $b = 9$, and $c = 4$.	Teacher led practice, then small group practice, focus on when to use law of sines versus cosines	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 3. I can find the area of oblique triangles using formulas.	SMP 5 Use appropriate tools strategically.	A realtor has measured a triangle lot that is 200 feet by 320 feet by 390 feet. What is the square footage of the lot that they will be selling?	Discovery of area formula, focus on when to use each of the two methods, small group practice	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p><input type="checkbox"/> 4. I can solve for perimeter and area in many different shapes.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find the area between a regular dodecagon and a circle that it is inscribed within that has a radius of 7 cm.</p>	<p>Connect to geometry vocabulary (especially review apothem), small group practice</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p><input checked="" type="checkbox"/> 5. I can solve for an angle or side length in a bearing word problem.</p>	<p>SMP 4 Model with mathematics.</p>	<p>Two airplanes leave an airport at the same time on different runways. One flies on a bearing of N66°W at 325 miles per hour. The other airplane flies on a bearing of S26°W at 300 miles per hour. How far apart will the airplanes be after 2 hours?</p>	<p>Connect to physics and oblique triangles, teacher model, then small group practice</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p><input type="checkbox"/> 6. I can convert between rectangular and polar coordinates.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Convert to polar (-5, -6).</p>	<p>Dynamic graphing, relate coordinate plane to polar plane, connect unit circle/coterminal angles, small group practice</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p><input type="checkbox"/> 7. I can convert equations between rectangular and polar form.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find the rectangular equation of $r = 1 + 2\sin\theta$.</p>	<p>Student discovery, connect single position to both systems, small group practice</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Key: Honors and CP Honors/CP optional Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>⊙ 8. I can graph basic polar equations.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Graph $r=2\sin\theta$.</p>	<p>Dynamic graphing, connect trig functions to polar coordinates, teacher led examples</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>⊙ 9. I can add, subtract, and multiply complex numbers in polar and rectangular form.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Convert to polar and multiply: $(-2 - 2i\sqrt{3})(4 - 4i)$.</p>	<p>Small group discovery, compare methods/results, list strategies as they relate to structures/operations</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>⊙ 10. I can solve for roots of complex numbers.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Find the fifth roots of $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$.</p>	<p>Connect DeMoivre's Theorem, student discovery, relate to graphing on complex plane</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>□ 11. I can sketch vectors, find component form, and find magnitude or direction of the vector.</p>	<p>SMP 5 Use appropriate tools strategically.</p>	<p>Given $C(2,3)$ and $D(9,1)$, express \vec{CD} in component form and \vec{CD}.</p>	<p>Relate multiple forms to one another (expression, geometric representation), dynamic graphing</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>□ 12. I can add vectors and find the resultant vector in</p>	<p>SMP 4 Model with mathematics.</p>	<p>Two tugboats pull a barge. The first tugboat pulls at a force of</p>	<p>Teacher led examples, connect diagram to structures of process,</p>	<p>Use of calculator, extended time, and specific other</p>

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>component form and it's direction and magnitude.</p>		<p>2500 lbs at 35° and the second tugboat pulls at a force of 2000 lbs at 355°. Find the direction and magnitude of the resultant vector.</p>	<p>accommodations/modifications per a student's IEP or 504 plan.</p>
<p>Benchmark Assessment</p> <ul style="list-style-type: none"> ● Not applicable 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 	
<p>Summative Assessment(s)</p> <ul style="list-style-type: none"> ● Additional Trig - Common Assessment ● Unit 8 Performance Task 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 	

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

Unit Title: Unit 9 Matrices	Timeframe/Pacing: 7 Days
Essential Questions <ul style="list-style-type: none"> ● How does a Matrix represent a collection of tabular data? ● Can we perform mathematical operations on Matrices? ● Can we use Matrices to solve problems? 	
Enduring Understandings <ul style="list-style-type: none"> ● A matrix contains a set of tabular data, with specific notation, operations, and algebraic properties. ● Matrices can help to solve problems such as solving (potentially very large) systems of equations. 	
Standards Taught and Assessed <ul style="list-style-type: none"> ● N-VM.C. Perform operations on matrices and use matrices in applications. 6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. 7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. 8. (+) Add, subtract, and multiply matrices of appropriate dimensions. 9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. 10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. 11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. 	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none"> ● Computer Science and Design Thinking: 8.1.12.DA.2: Describe the trade-offs in how and where data is organized and stored. ● Computer Science and Design Thinking: 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena. 	
Highlighted Career Ready Practices and 21st Century Themes and Skill <ul style="list-style-type: none"> ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions. ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 	

Key: ■ Honors and CP □ Honors/CP optional ◎ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>Social Emotional Learning Competencies</p> <ul style="list-style-type: none"> 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations. 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity} 				
<p>Pre-Assessment</p> <ul style="list-style-type: none"> Solve a linear system of equations (2 variables) 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
<p>■ 1. Perform matrix addition, subtraction, scalar multiplication operations.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Given matrices A and B, find $A + B$, $A - B$, $A + 2B$, or explain why not possible</p>	<p>Using textbook, videos, teacher prepared notes, and examples.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 2. Perform (full) matrix multiplication operation</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Given matrices A and B, find $A \times B$ and $B \times A$, or explain why not possible. Are these the same?</p>	<p>Using textbook, videos, teacher prepared notes, and examples.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>■ 3. Find the inverse of a 2x2 dimension matrix</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Given 2x2 matrix A, find inverse A, or explain why not possible.</p>	<p>Using textbook, videos, teacher prepared notes, and examples.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 4. Represent a linear system of equations as a matrix equation</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Write a linear system of equations in matrix form.</p>	<p>Using textbook, videos, teacher prepared notes, and examples.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>■ 5. Solve a matrix equation using the inverse matrix, and show this can be used to solve a linear system</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Given a 2x2 matrix equation of the form $AX=B$, solve by finding inverse A.</p>	<p>Using textbook, videos, teacher prepared notes, and examples.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.</p>
<p>Benchmark Assessment</p> <ul style="list-style-type: none"> ● Not applicable 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p>		
<p>Summative Assessment(s)</p> <ul style="list-style-type: none"> ● Matrices - Common Assessment ● Unit 9 Performance Task: Solve a very large system of equations with matrices using technology. ● Unit 9 Performance Task: Coding Messages using Matrices 		<ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum

Unit Title: Unit 10 Conic Sections	Timeframe/Pacing: 15 Days
Essential Questions <ul style="list-style-type: none"> ● Similar to a circle, can we find other intersections formed by a plane passing through a cone? ● What are the algebraic representations of these other figures or relationships? ● Can we recognize a conic section from the algebraic representation? 	
Enduring Understandings <ul style="list-style-type: none"> ● Conic sections are a set of classically studied relationships, with interesting geometric and algebraic properties. ● Conic sections appear in many real world situations and understanding them can help to solve problems. 	
Standards Taught and Assessed <ul style="list-style-type: none"> ● G-GPE-A. Translate between the geometric description and the equation for a conic section. <ol style="list-style-type: none"> 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. 2. Derive the equation of a parabola given a focus and directrix. 3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. 	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none"> ● Science: HS-ESS1-4: Earth and the Solar System. Kepler’s laws describe common features of the motions of orbiting objects, including their elliptical paths around the sun. Orbits may change due to the gravitational effects from, or collisions with, other objects in the solar system. 	
Highlighted Career Ready Practices and 21st Century Themes and Skill <ul style="list-style-type: none"> ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 	
Social Emotional Learning Competencies <ul style="list-style-type: none"> ● 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations. ● 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity} 	

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

Pre-Assessment <ul style="list-style-type: none"> Distance and midpoint formulas 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
<p>■ 1. I can graph and write an equation for a circle.</p>	SMP 7 Look for and make use of structure.	Write the equation for the circle with radius 5 and center (2, -1).	Recall equation from Geometry. Recall and practice completing the square from Algebra 2	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
<p>■ 2. I can graph and identify important parts of an ellipse.</p>	SMP 7 Look for and make use of structure.	Graph the ellipse $4(x - 1)^2 + 9(y + 2)^2 = 36$.	Small group activity, students sketch ellipse using string attached to foci.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
<p>■ 3. I can write the equation of an ellipse given information.</p>	SMP 7 Look for and make use of structure.	Write the equation of the ellipse with vertices at (3, -2) and (-3, -2) and	Using a labeled example, note specific values and write equations.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

		co-vertices of (0, 4) and (0, -8).		504 plan.
■ 4. I can graph and identify important parts of a hyperbola.	SMP 7 Look for and make use of structure.	Graph, find the foci, and the asymptotes. $\frac{x^2}{25} - \frac{y^2}{16} = 1$	Using the standard form of the equation, draw the graph.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 5. I can write the equation of a hyperbola given information.	SMP 7 Look for and make use of structure.	Find the standard form of the equation of each hyperbola where the center is (-2, 1), a focus is (-2, 6), and a vertex is (-2, 4).	Using a labeled example, note specific values and write equations.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 6. I can graph and identify important parts of a parabola.	SMP 7 Look for and make use of structure.	Find the focus and directrix of the parabola given by $y^2 = 12x$. Then graph the parabola.	Using the standard form of the equation, draw the graph.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 7. I can write the equation of a parabola given information.	SMP 7 Look for and make use of structure.	Find the standard form of the equation of a parabola with focus (5, 0) and directrix $x = -5$.	Using a labeled example, note specific values and write equations.	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.

Key: ■ Honors and CP □ Honors/CP optional ○ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>■ 8. I can determine the type of conic section from an equation.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Determine the type of conic section. $4x^2 - 9y^2 - 8x - 36y - 68 = 0$</p>	<p>Consider coefficients, play “name the mystery” conic.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student’s IEP or 504 plan.</p>
<p>9. I can solve for points of intersection between conic sections and lines. ■</p>	<p>SMP 6. Attend to precision.</p>	<p>Solve for the points of intersection. $\begin{cases} 2x + y = 3 \\ x^2 + y^2 = 16 \end{cases}$</p>	<p>Recall solution methods from Algebra 2. Consider positive and negative solutions. .</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student’s IEP or 504 plan.</p>
<p>⊙ 10. I can rotate the axis of a conic section.</p>	<p>SMP 7 Look for and make use of structure.</p>	<p>Rewrite equation (below) in a rotated $x'y'$ system. $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$</p>	<p>Using online textbook materials, videos, teacher prepared notes, and exercises.</p>	<p>Use of calculator, extended time, and specific other accommodations/modifications per a student’s IEP or 504 plan.</p>
<p>Benchmark Assessment</p> <ul style="list-style-type: none"> ● Not applicable 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p>		
<p>Summative Assessment(s)</p> <ul style="list-style-type: none"> ● Unit 10 Conic Sections - Common Assessment ● Unit 10 Performance Task 		<ul style="list-style-type: none"> ● Use of calculator, extended time, and specific other accommodations/modifications per a student’s IEP or 504 plan. 		

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum

Unit Title: Unit 11 Sequences and Series	Timeframe/Pacing: 10 days
<p>Essential Questions</p> <ul style="list-style-type: none"> ● How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations? ● What makes an algebraic algorithm both effective and efficient? ● How can change be best represented mathematically? 	
<p>Enduring Understandings</p> <ul style="list-style-type: none"> ● Algebraic representation can be used to generalize patterns and relationships. ● Algebraic and numeric procedures are interconnected and build on one other to produce a coherent whole. ● The symbolic language of algebra is used to communicate and generalize the pattern in mathematics. 	
<p>Standards Taught and Assessed</p> <ul style="list-style-type: none"> ● F-IF. A Understand the concept of a function and use function notation. <ol style="list-style-type: none"> 3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. ● F-BF A. Build a function that models a relationship between two quantities. <ol style="list-style-type: none"> 2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ● A-SSE B. Write expressions in equivalent forms to solve problems. <ol style="list-style-type: none"> 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. 	
<p>Highlighted Interdisciplinary Connections</p> <ul style="list-style-type: none"> ● Science: SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one on- one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively. 	
<p>Highlighted Career Ready Practices and 21st Century Themes and Skill</p> <ul style="list-style-type: none"> ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 	

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<p>Social Emotional Learning Competencies</p> <ul style="list-style-type: none"> 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations. 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity} 				
<p>Pre-Assessment</p> <ul style="list-style-type: none"> Find patterns and predict future numbers 		<p>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</p> <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
■ 1. I can write an explicit equation for arithmetic sequences.	SMP 7 Look for and make use of structure.	Write the explicit equation for the arithmetic sequence where the second term is 20 and the tenth term is 44.	Connect to linear functions, students utilize formula	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 2. I can write an explicit equation for geometric sequences.	SMP 7 Look for and make use of structure.	Write the explicit equation for the geometric sequence below: 2, 6, 18, ...	Connect to exponential functions, students utilize formula	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 3. I can find missing	SMP 7 Look for and	Find the 8th term of the	Students discuss	Use of calculator, extended

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

terms for geometric and arithmetic sequences.	make use of structure.	sequence: 0.008, 0.16, 3.2, 64, ...	strategies, students utilize formula	time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 4. I can simplify factorial expressions.	SMP 7 Look for and make use of structure.	Simplify as much as possible: $\frac{(n+2)!}{2n!}$	Relate to simplifying exponential expressions, small group practice	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 5. I can find sums of arithmetic and geometric series (if it exists) for infinite and finite series.	SMP 7 Look for and make use of structure.	Find the sum of the first 10 terms of the sequence below: 4, -8, 16, -32, 64, ...	Students discuss strategies, small group practice	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 6. I can write a series in summation notation and evaluate.	SMP 7 Look for and make use of structure.	Express the sum using summation notation: $3+6+9+\dots+21$.	Discuss structure, small group practice, challenge: create multiple summation notations to represent the same series	
■ 7. I can use sequences or series to solve application problems.	SMP 4 Model with mathematics.	As part of her retirement savings plan, Simone deposited \$4000 into a bank account during her first year in the workforce. During each subsequent year, she deposited \$212 more than the previous	Small group practice, connect to idea of exponential growth	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

		year. Find the total amount she deposited in 25 years of work.		
■ 8. I can use recursive formulas to express how to find the next term in a sequence.	SMP 8 Look for and express regularity in repeated reasoning.	Find a recursive formula to represent the total amount of money Simone has earned since she has started working.	Use Fibonacci, small group practice	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
□ 9. I can use induction to prove the sums of different series. (if time allows)	SMP 2 Reason abstractly and quantitatively.	Prove the sum of the first n powers of 2 is $2^n - 1$.	Large discussion, focus on how if it works for one and then the next one works how that pattern can continue	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
⊙ 10. I can find the limit of a sequence.	SMP 2 Reason abstractly and quantitatively.	Find $\lim_{x \rightarrow \infty} \frac{n}{n+1}$.	Large discussion, focus on what we think the last term would be and what happens when you plug in large numbers	
Benchmark Assessment <ul style="list-style-type: none"> Not applicable 		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Summative Assessment(s) <ul style="list-style-type: none"> Sequences and Series - Common Assessment Unit 11 Performance Task 				

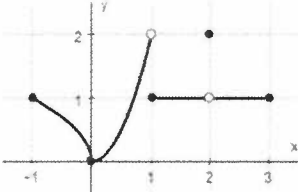
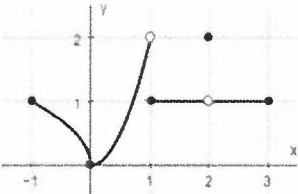
Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

Unit Title: Unit 12 Limits	Timeframe/Pacing: 13 days
Essential Questions <ul style="list-style-type: none"> ● How can I describe the behavior of a function? ● How do mathematical ideas interconnect and build on one another to produce a coherent whole? 	
Enduring Understandings <ul style="list-style-type: none"> ● Behavior and value of a function may differ. ● Numeric fluency includes both the understanding of and the ability to appropriately use numbers. 	
Standards Taught and Assessed <ul style="list-style-type: none"> ● F-IF.7d (+) - Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and show end behavior. 	
Highlighted Interdisciplinary Connections <ul style="list-style-type: none"> ● Science: SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one on- one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively. 	
Highlighted Career Ready Practices and 21st Century Themes and Skill <ul style="list-style-type: none"> ● 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions ● 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 	
Social Emotional Learning Competencies <ul style="list-style-type: none"> ● 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations. ● 2.2.12.LF.4: Exhibit responsible social behavior by including and cooperating with classmates of all skill levels, assisting when needed, and collaborating respectfully to solve problems in groups, teams, and in pairs during physical activity {or mathematical activity} 	
Pre-Assessment <ul style="list-style-type: none"> ● Simplify expressions 	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)

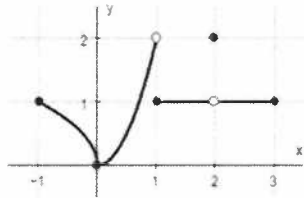
Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

<ul style="list-style-type: none"> Piecewise functions 		<ul style="list-style-type: none"> Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan. 		
Student Learning Objectives	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
<p>■ 1. I am able to interpret limit notation and determine if a limit exists.</p>	SMP 7 Look for and make use of structure	<p>Does the limit of the function graphed below exist at $x=1$? Explain your answer.</p> 	Teacher led discussion, comparing relation amongst platforms (graph, equation, table), varied examples amongst small group	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
<p>■ 2. I am able to find limit the limit of a function using tables and graphs.</p>	SMP 3 Construct viable arguments and critique the reasoning of others.	<p>Evaluate $\lim_{x \rightarrow 2} f(x)$ where $f(x)$ is graphed below:</p> 	Student discovery example, comparing strategies/solutions in conversation, dynamic graphing with exercises	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
<p>■ 3. I am able to find limits using properties of limits.</p>	SMP 7 Look for and make use of structure.	<p>Evaluate $\lim_{x \rightarrow 3} \frac{3x+1}{2x+5} - 2x$</p>	Scaled exercises, student conversation in small group, teacher led discussion	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or

Key: ■ Honors and CP □ Honors/CP optional © Honors

**Hillsborough Township Public Schools
Math Analysis Mathematics Curriculum**

				504 plan.
■ 4. I am able to find limits using algebraic methods.	SMP 7 Look for and make use of structure.	Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{x+4}-2}{x}$	Student led conversations of strategies/procedures, connect previous definitions and practices to limits, scaled examples	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
■ 5. I am able to determine whether a function is continuous at a specified point.	SMP 3 Construct viable arguments and critique the reasoning of others.	Is the function graphed below continuous at $x=2$? Explain your reasoning. 	Student investigation/discovery, small group exercises, dynamic graphing	Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment <ul style="list-style-type: none">End of Course Cumulative Assessment		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none">Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.		
Summative Assessment(s) <ul style="list-style-type: none">Limits- Common AssessmentUnit 12 Performance Task		Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504) <ul style="list-style-type: none">Use of calculator, extended time, and specific other accommodations/modifications per a student's IEP or 504 plan.		

Key: ■ Honors and CP □ Honors/CP optional ⊙ Honors

Bibliography
Math Analysis

Digital Textbook Resource:

Blitzer, R. (2018). *Precalculus*. Upper Saddle River, New Jersey: Pearson Prentice Hall.

Digital Resources:

www.mymathlabforschool.com